第 17 卷　第 7 期
2009 年 7 月

光学 精密工程
Optics and Precision Engineering

Vol. 17　No. 7
Jul. 2009

文章编号　1004-924X(2009)07-1630-05

时钟抖动下加速度测量值的数值双积分误差

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摘要：采用蒙特卡洛分析法，对由于时钟抖动造成的加速度计定位误差进行了系统的分析。仿真结果表明，在存在时钟抖动的情况下，加速度测量值是渐近服从高斯分布的，这个结果与理论分析相吻合。对积分距离的方差与时钟抖动之间的关系进行了分析，理论及仿真分析结果都表明了噪声的数值双积分的积分误差与时钟抖动的大小成正比。当输入一个幅值为 1g，周期为 1 s 的正弦波加速度信号时，1 s 后数值双积分的积分误差与时钟抖动的比例系数为 0.261 1。

关　键　词：微加速度计；时钟抖动；数值双积分；微机电系统
中图分类号：TH824.4　　**文献标识码：**A

Error analysis on numerical double integration in acceleration measurement with clock jitter

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Abstract：The positioning error of accelerometers due to a clock jitter is analyzed by Monte Carlo analysis method. The simulation results reveal that the distribution of acceleration measurements with the clock jitter is approximately Gaussian distribution，which is well coincident with the theoretical analysis. Moreover，the relation between the variance of integration distance and the clock jitter is analyzed. Both the theoretical and simulation results indicate that the error of numerical double integration due to the clock jitter increases proportional to the clock jitter. When the input acceleration signal is a sine wave with a amplitude of 1g and a period of 1 s，the ratio between the integration error of numerical double integration and the clock jitter is 0.261 1 after 1 s.

Key words：micro-accelerometer；clock jitter；numerical double integration；MEMS

1　Introduction

Accelerometers have undergone a technological evolution and some of them are fabricated by advanced surface micromachining techniques which allow them to sense in one，two or three axes[1]. Recently，the accelerometer have become a popular device for the safety requirement in automobiles，especially in seat belts and airbag systems. This leads to a large demand for low-cost and small-size accelerometers capable of

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收稿日期：2009-01-30；**修订日期：**2009-04-30.
基金项目：Supported by the National Hi-Tech Research and Development Program of China(863 Program)(Grant No. 2008AA042201)

sensing up to 50 *g*. Thus, integrated capacitive-based accelerometers that meet the requirements are well received. To reduce the probability of accidents due to the malfunction under interferences such as electromagnetic and pulse noises, air-bag restraint systems require accelerometers not only to have the characteristics of fast response, high sensitivity, low cost and good reliability but also to resist any outside disturbances[2].

With the development of the technology, high-performance micro-machined accelerometers have been widely used in biomedical applications, navigation systems, and seismometry for oil explorations[3-4]. In these cases, information is required on the full acceleration vector. In this peper, we exploit the advantages of a closed-loop control strategy to increase the dynamic range, linearity, and the bandwidth of sensors to satisfy the application requirements. To decrease power consumption, capacitive devices are often preferred[5-6].

When the accelerometers are used to position, the acceleration will be sampled by an analog-to-digital (A/D) converter based on a clock generation. Then the measurements would be used by numerical double integration to calculate the position. However, due to clock jitter, there is always sampling error. In this paper, the error of calculated measurement will be analyzed.

2 Relation between clock jitter and phase noise with white noise

Defining a jitter has been a troublesome task since there is no universal accepted terminology for the jitter, such as the absolute jitter, cycle jitter, cycle-to-cycle jitter, long term jitter, edge-to-edge jitter, period jitter, and the tracking jitter[7]. In this paper, the concept of period jitter will be adopted. If $\phi_n(t)$ is the phase of a differential ring oscillator at time t, and $S_\phi(f)$ is the power spectral density of $\phi(t)$[8]. Then

$$\tau_i = \frac{1}{2\pi f_0}(\phi(t_{i+1}) - \phi(t_i)) = \frac{1}{2\pi f_0}\Delta\phi_i\ , \tag{1}$$

where $t_{i+1} - t_i = T_0 = 1/f_0$, T_0 and f_0 are the period and frequency of the oscillator, respectively.

So the power spectral density of the phase deviation τ can be derived as follow:

$$S_{\Delta\phi}(f) = S_\phi(f)\ |1 - \mathrm{e}^{-\mathrm{j}2\pi f/f_0}|^2 = 4S_\phi(f)\sin^2(\pi f/f_0)\ . \tag{2}$$

Then the spectral density of the jitter is given by

$$S_\tau(f) = S_\phi(f)\,\frac{\sin^2(\pi f/f_0)}{(\pi f_0)^2}\ . \tag{3}$$

If the phase $\phi(t)$ is a stochastic stationary process, then we can use the Wiener-Khinchine theorem to calculate the mean square value σ_τ^2 by[3-4]

$$\sigma_\tau^2 = \int_0^\infty S_\tau(f)\,\mathrm{d}f = \int_0^\infty S_\phi(f)\,\frac{\sin^2(\pi f/f_0)}{(\pi f_0)^2}\,\mathrm{d}f\ . \tag{4}$$

This is the relation between the phase noise spectral density and the jitter.

In this paper, the positioning error due to clock jitter will be presented. If the A/D converter uses ring oscillator as a clock generator, the macro noise source feeding the integrator is given by

$$S_\phi(f) = \frac{2f_0^2 c_w}{f^2}\ . \tag{5}$$

Then the period jitter can be expressed by

$$\sigma_t^2 = c_w T_0. \tag{6}$$

3 Relation between error of double integration and clock jitter

If the input acceleration signal is sine wave with frequency of 1 Hz and a magnitude of

1 $\mathrm{m/s^2}$, the input acceleration signal can be expressed as

$$a=\sin(2\pi t)\ . \tag{7}$$

Therefore we can easily calculate the distance of the object which has moved 1 s, if the initial location is zero and the initial speed is also zero. Using simple calculation, the distance can be computed as

$$d=\left[-\frac{1}{(2\pi)^2}\sin(2\pi t)+\frac{1}{2\pi}t\right]\Bigg|_0^1=\frac{1}{2\pi}\ . \tag{8}$$

However, the acceleration measures are discrete signals. So when using the A/D converter to measure the acceleration signals, we can only obtain the acceleration values at discrete time points. In this paper, the sampling period of the simulation is $T=1\times10^{-4}$ s. If the input acceleration signal is the same as the mentioned above, we can obtain the discrete acceleration at time nT which can be expressed as $a[n]$. Then the position can be estimated from accelerometer measurements by a numerical double integration of the accelerometer signal. Using rectangular numerical integration formula, the distance of an object between its stating point and its position at time NT can be expressed as[9]

$$s[N]=T^2\sum_{n=1}^{N}(N-n)a[n]\ . \tag{9}$$

Theoretically, if the sampling period is small enough, the error of numerical double integration can be ignored. However, if there is clock jitter of the sampling clock, the error of numerical double integration can not be ignored though the sampling frequency is high enough.

In order to analyze the measurement error, Monte Carlo analysis method will be presented here. The simulated results of acceleration measurements in 1 s are shown in Fig. 1. The fitting curve implies that the distribution of acceleration measurements is approximately Gausses distribution.

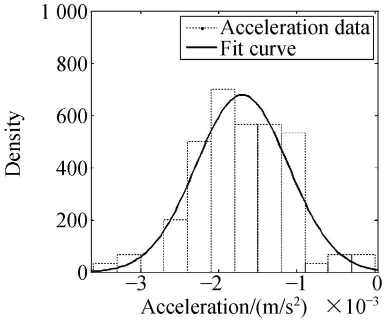

Fig. 1　Acceleration measurement probability density and its normal distribution fitting curve

In order to deduce the relationship between the measurement errors and the clock jitter, the first order Taylor expanding expression will be used. Thereby the approximate formula could be expressed as

$$\sigma_a^2=|\,a'(t)\,|^2\sigma_t^2=|\,2\pi\cos(2\pi t)\,|^2c_w t\ . \tag{10}$$

The simulated and theoretical results are compared in Fig. 2. As Fig. 2 shown, the theoretical results are in accordance with the simulated results.

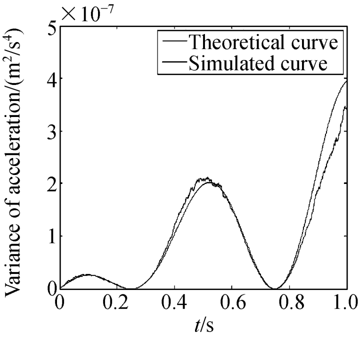

Fig. 2　Variance of acceleration due to clock jitter in time domain

Using equation (1), the distance from the acceleration measurements could be calculated. Based on the Monte Carlo analysis method, the simulation will be repeated stochastic 100 times when $c_w=1\times10^{-8}$ s. The probability density of the simulation results is shown in Fig. 3. The distribution of the calculated distance could also be approximated by Gausses distribution.

These above results imply that the errors of acceleration measurement and the calculated distance can be approximated by white noise. It means that most of the errors can be removed by denoising, such as wavelet shrinkage. Fig. 4 shows the relation between the variance of calculated distance and the magnitude of clock jitter. The result implies that the variance is in proportion to clock jitter. The proportional coefficient obtained by curve fitting is 0. 261 1.

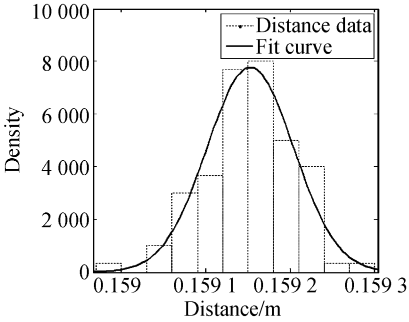

Fig. 3　Calculated distance probability density and its normal distribution fitting curve

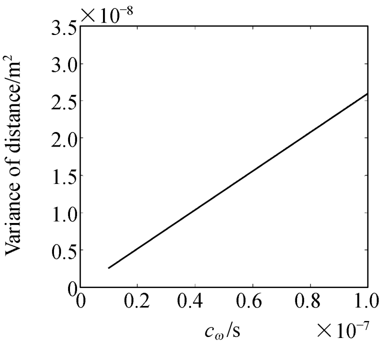

Fig. 4　Variance of distance due to clock jitter

4　Conclusions

In this paper, the positioning error due to the clock jitter has been analyzed by Monte Carlo method. Both the theoretical and the simulation results reveal that the calculated error of numerical double integration due to the clock jitter increases proportional to the clock jitter. So if the accelerometer is applied to a rigorous positioning system, the clock jitter must be depressed.

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●下期预告

迈氏腔光纤激光器的相干合成

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为了获得高功率和高亮度光纤激光输出，利用一块45°半透半反分束镜作为干涉元件，构建迈氏腔结构，使两路光纤激光器在输出端相干相长，从而实现相干合成输出的方案。理论分析了此方案的锁相机理，并在实验中成功实现了两路光纤激光器的相位锁定，使合成输出激光在自由空间传播，获得了功率约360 mW的相干合成激光输出，功率合成效率约为73%，合成输出后的激光束在轴上光强获得了大幅度提高。本方案结构简单，利用了光纤耦合器干涉锁相的相同原理，但却避免了光纤耦合器承受功率低的缺点，所有元件均可承受高功率，因此可实现更高功率的相干合成激光输出，是一种有前途的高功率光纤激光器相干合成方案。